Boolean Competition

“Not one of them was capable of lying,
There was not one which knew that it was dying
Or could have a rhythm or a rhyme,
Assumed responsibility for time.”

W.H. Auden
In the market-as-a-game, decoding strategy means that competition is a process and that the game is necessary because of its function as a determination of evolving market systems. The process could be described as efficient contracting between firms. To put it another way, competition in an evolving market system can be viewed as an assignment of property rights to the market system rents (McNutt, 2005). Management should enquire whether the \((p,q)\) pair is a strategic outcome, and whether it is stable in an evolving market system. In other words, will \((p,q)\) persist, and if a firm chooses their actions from that pair, will the choice of price and quantity continue to distribute rents to the firm? In an evolutionary game setting, being able to choose may well turn out to be a disadvantage (Maynard Smith, 1982). This is tantamount to there being no choice for the firm.

**Market Systems**

Game embedded strategy (GEMS) works on the assumption that there is a Boolean network of firm behaviour. At its simplest, a Boolean network provides for a decision, which stipulates that each firm in a market with \(n\) firms acts (say, on price or on R&D expenditure or on innovation) if, and only if, two or more firms act. For example, firm A makes a decision if two other firms make a decision, but each firm can decide to act or not at each decision point.

While this can be generalised to explain the quintessence of aggressive competition, its central message is that for a random firm in an evolving market system its behaviour on price and quantity may be fixed exogenously. So, for example, a random firm offers \(n\) products and for each feasible bundle \(q\) it charges a price \(p(q)\). Profit is strictly a function of the prices. Each customer responds by choosing its preferred bundle and paying the price. The firm’s objective in choosing the price is to maximise its profit contribution, obtained as the difference between the revenues collected and the costs it incurs to supply the bundles demanded. The firm incurs this cost only if a customer purchases a non-negative amount of some product. The prices evolve as marginal prices
(Wilson, 1991), since they are observed as partial sums of the associated prices for incremental bundles of product.

**Network Effects**

The standard economic model of collusion begins with the assumption that cartels are costless to form and maintain. But cartels accrue costs, for example the costs of monitoring cheats. Johnsen, in a classic article on the assignment of property rights to cartel rents, argued that “maximising cartel wealth translates into maximising the discounted value of the difference between gross cartel rents and cartel enforcement costs” (Johnsen, 1991, p. 189). The closer the cartel comes to the \((p,q)\) that maximises gross cartel rents, the greater the incentive for members to cheat.

The key to applying the concept of tacit collusion is to distinguish tacit collusion from competitive aggressive behaviour. In any defence of an alleged price-fixing case, the mere fact of adherence to prices may not establish an agreement to adhere to them (McNutt, 2003). Therefore, it would be unlikely for a competition court to find that adherence alone could prove, beyond all reasonable doubt, an agreement to adhere to prices; there may be ‘an asymptote in prices’ (ASP) observed as an accidental sameness in price across the market.

In Figure 10.1, market shares are denoted by \(s\). Introduce a zero-sum assumption, which implies that as the market share of firm 1 \((S_1)\) increases, the market share of firm 2 \((S_2)\) decreases, as noted in the left-hand quadrant. This would represent a classic case of real competition characterised by competitive aggressive behaviour through loss in market share. The history of market price is represented by the function \(P_i(S_i)\), which is asymptotic to a lower bound, \(P = LMC\), the long-run competitive price for the market in which these two firms interact. The loss in market shares accruing to any one firm is represented by \(\Delta_w\).

The Boolean network provides for a competitive zero-sum rule, \(\Delta_w x_i\), as follows:

\[
\Delta_w x_i = x_i - w_i = \{(1, -1), i = 1 \& (-1, 1), i = 2\}
\]
Markets can be either collusive or competitive. But firms in both situations behave rationally and independently. However, tacit collusion needs some form of an enforcement mechanism to sustain a coordinated equilibrium at f in Figure 10.1. In the absence of such a credible mechanism, the market price at f could be described as the price at a point in the history of the market moving towards a long-run competitive price. It is the market characteristics and the history of prices therein that make a market more collusive and less competitive. An ASP price standard is asymptotically close to a long-run competitive price. It therefore follows that not all instances of parallel behaviour could give rise to the same strength of inference that the parallelism results from anything other than the independent commercial judgement of the firms in a Boolean network.

**Price Coordination**

Economic theory would have us believe that price coordination is designed to reduce the uncertainty associated with interdependence and
thereby decrease the likelihood of mutually destructive price competition. Consider n-player rent and profits games. The player set, the set of firms \{1, 2, 3, \ldots n\}, is denoted by \(N\), the set of coalitions by \(2^N\). Such a game is a real valued function \(v: 2^N \to \mathbb{R}\) with \(v(0)\), which assigns to each coalition, \(S\), its worth \(v(S)\). The worth \(v(S)\) can be interpreted as the reward, which the players in \(S\) can obtain by working together. We denote the set of n-person rent and profits games by \(G_n\). The main problem in cooperative game theory for analysts is how to divide \(v(N)\) among the players if the grand coalition forms.

There are many solutions, and they offer competition policy economists and lawyers a new tool of analysis and set of defences respectively: in some games, for example, the dividend of any one player is proportional to the marginal contributions of the players to the grand coalition (Owen, 1982). The marginal contributions are captured by movements along the \(P_i(S)\) function in Figure 10.2. This is analogous to the biological concept of ‘carrying capacity’, that is, the maximum number of firms that can be sustained by a given amount of resources. It is not unlike an optimal club size in the provision of public goods (McNutt, 2002). Later in this chapter, we include this as the parameter \(K\) in the equations for contest competition. Intuitively, one knows that not every firm can adapt to external threats, and with the passage of time only a few large firms will survive.

Perfect information amongst businesses would allow some to quickly enter the price-fixed markets and compete away the supra-competitive profits. The competition would soon drive prices down to only an insignificant fraction above the competitive level (Averitt and Lande, 1997). As the industrial (business) stage in different jurisdictions assumes the status of an oligopoly, there may be increasing support for the argument that a measure of price coordination is necessary in an oligopolistically structured industry. Innovation may require firms to enter complex contracts and relationships with other firms in order to bring technology to the economic market. But uncertainty is high. The uncertainty is especially high for the development and commercialisation of new technology. Accordingly, innovating firms may need to achieve greater coordination than the price system alone appears to be able to bring about (Jorde and Teece, 1990).
Scramble, Combat and Contest

Imagine that there are \( n \) operating firms in a market that is viable for only \( m < n \). This may, for example, be the case in markets wherein technology or innovation involves a fixed cost of production. Selection may then operate in order for the firms not to lose money. Fudenberg and Tirole (1986) examine how the remaining firms are picked in order to explain why selection is not immediate, that is, why there are periods of time over which firms lose money but do not leave the market. They find it difficult to pin down a stable equilibrium outcome and conclude that there is a strictly positive but possibly small probability that the firm, as a player in a game, ‘enjoys fighting’.

To put it another way, a firm with strictly positive duopoly profit never drops out of a market — staying in the market is a dominant strategy. Biologists have also analysed this type of situation. For example, animals may spend time or energy in a seemingly useless fight for prey (Maynard Smith, 1974). Firms may persist in a market. Therefore, firm 1, if it observes that firm 2 is still in at time \( t \), ought to infer that firm 2 has a positive duopoly profit, and therefore will not drop out. Then firm 1, if it has a negative duopoly profit, ought to leave.

Moore’s ‘Form of Friction’

The fluctuation in the population of firms describes the process of competition. Much earlier in the history of economics, at a time when the Principles of Marshall were the subject of debate amongst Sraffa (1926) and Hotelling (1929), who laid the intellectual foundations for the concept of imperfection in the market, Moore (1906) offered a critical observation that has influenced the discussion in this chapter. Writing in the Quarterly Journal of Economics, Moore (1906) asked: “What is the nature of the limitation of the applicability of propositions under the hypothesis of perfect competition? The almost invariable answer to this question is that the imperfection of competition is simply a form of friction, producing for the most part, a negligible variation from the standards that prevail in a regime of perfect competition” (p. 211, italics added).
At the turn of the 20th century, the rigours of biology and physics were available to aspiring economists intent on developing an intellectual foundation for economics. Marshall opted for the rigours of physics in writing his *Principles* rather than the mathematical modelling of ecological systems. May (1973) commented that such models aim to provide a conceptual framework for the discussion of broad classes of phenomena. But Moore was asking a question, which could be answered only in the context of understanding the evolution of firms and markets. In trying to adapt what Moore may have meant by ‘friction’, we introduce types of competition adapted from models for species competition in biology (Hassell, 1976).

**Types of Competition**

One type of competition is **scramble competition**, wherein there is an exact equal partitioning of the market and hence an equal division of the effects of competition between the competitors. Scramble may be manifested by changes in the size of firms or number of firms. A second type of competition is **combat competition**, where the acquisition of market share requires constant defence. This would be characteristic of a more stable market system. Combat competition may be about entry at the margin and would manifest itself when an increasing number of firms is not an advantage to the market system. And finally, a third type of competition is **contest competition**, where the market is unequally partitioned in that some firms are content with their market shares while other firms are targets of merger or takeover.

Contest would occur, for example, where individual firms compete either for a given market share or for market position. Contest competition can be seen as a mechanism that tends to maintain the market level of concentration as long as the number of firms does not change. One essential characteristic of contest and scramble is that in both cases there is no exit of firms below a threshold level of concentration when there is ample market share for all competing firms. Above this threshold level of concentration, exit increases abruptly in the case of perfect scramble but gradually in the case of perfect contest. This follows necessarily from
the requirement that contest leaves a constant number of firms in the market system.

It is conceivable that all competition under normal market conditions could fall between these idealised extremes of contest and scramble. Pure contest is rare; as individual firms compete for market share, there will often be a compromise as combat competition becomes more intense and average market share may be reduced. Larger firms will evolve and survive as they can provide economies of scale in production and innovation. Large firm size becomes essential to the success of innovative activity. With economies of scale, large firms make available sufficient resources for new innovative activity.

This process is not dissimilar to Schumpeter’s cycle of ‘creative destruction’ in which old industrial structures, their products, their manufacturing processes and their organisational form are continually changed by new innovative activity. Schumpeter’s (1934) original hypothesis was that economic growth occurs through a process of ‘creative destruction’ and that long-term growth is intricately linked with innovation. The introduction of a new good or new quality of an existing good, the introduction of a new method of technology, the introduction of a new organisational form or the opening of a new market are all characteristics of a market system.

In understanding how a market system differs from a market structure, we need to acknowledge that firms in a market structure cannot easily adapt internally to an external threat. The classic monopoly firm has little or no incentive to change if the status quo is profitable. Incumbents may attempt to retard entry. The classic monopoly and the incumbent both suffer from a ‘box-ticking exercise’ whereby behaviour and conduct is predetermined by the structure of the market. It is as if the very structure of the market creates what Nolan and Croson (1995) called ‘structural inertia’. In contrast, firms evolve in a market system, making radical changes in both strategy and organisation in the face of external threats. The s-firm, for example, is an internal response from the workers and management to the external threat of unemployment.
Modelling Contest Competition

The specification of the entry function and its concavity in Chapter 6 highlighted the restrictive nature of entry. E(q) translated into an actual market share if entry was impeded. In a market system, all firms (potential entrants and incumbents) have the potential to grow exponentially, as expressed by the system equation

$$\frac{dn}{dt} = rn$$

Thus, the rate of change in the number of firms, \(n\), with the passage of time, \(t\), is the product of the numbers of firms and their intrinsic rate of natural increase, \(r\). This is the maximum instantaneous rate of increase under the Scherer and Ross (1990) conditions for competition. To find the number of firms at any given time \(t\), we integrate to get

$$n_t = n_0 e^{rt}$$

where \(n_0\) is the number of firms at time \(t_0\). From this, we can plot the exponential growth of the number of firms with time. However, no firms can sustain such an increase for long. Competition for resources will become increasingly more acute and the net rate of increase \([dn/dt]\) reduced, either due to mergers or acquisitions, exit of firms or both.

Therefore, the market system can be described as

$$\frac{dn}{dt} = r.n \left\{\left(\frac{\beta - n}{\beta}\right)\right\}$$

where \(\beta\) is the ‘carrying capacity’ of the system at a point in time – the maximum number of firms that can be sustained by a given amount of limited resources. If we use the firms’ production levels at \(t \geq 0\) as a proxy for the amount of limited resources, the carrying capacity could be defined as

$$CC = (\text{firm’s production levels})^{-n} / \left\{\frac{n}{n + 1}\right\}^n$$

The emphasis is on the available carrying capacity at \(t \geq 0\). Knowledge of rising scale economies may not be known to the fact
finder at \( t = t_0 \). The number of firms may fall as firms merge to gain efficiencies. The available carrying capacity is an important part of the determination of whether a merger may lead to a dominant position or whether a given market share level may converge towards a collusive outcome. Using integration, we have

\[
    n_t = \frac{\beta}{1 + q e^{-r t}}
\]

where \( q = \frac{\beta}{2} \) is the point of inflexion on the time axis as illustrated in Figure 10.2, adapted from Varley (1973). The growth in the number of firms can be described as sigmoid. It commences almost exponentially, but as the number of firms increases there is more and more feedback from the term \( (\beta - n)/\beta \), representing the effects of increased competition. A rapidly growing firm in contest competition may be more likely to make a horizontal acquisition because it would

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**Figure 10.2**

Contest Competition

![Diagram showing contest competition with points A and B, q = β/2, and time intervals t_1, t_2, t_3.](image-url)
be better able to use the additional capacity. The net rate of increase thereof declines until, when the carrying capacity is reached, \( n_t = \beta \), there is no further change in the number of firms. And the market system is therefore at equilibrium, \( n^* = \beta \).

The model offers a stable equilibrium since the number of firms will always return to its equilibrium following an external threat or disturbance. For example, a declining firm operating under diseconomies of scale will be more likely to sell and exit the system. Tremblay (1987) found that a firm with large economies of scale will have a greater incentive to merge if the economies are multi-plant in nature. The essential economic character of what is observed at \( t \geq 0 \) is one of contest competition; the maximum number of firms is independent of the initial density of firms.

**Spherical Competitors**

The basic conditions of the market system, the carrying capacity, can plausibly explain the evolution of the system. The litmus test is whether the fact finder can define the market within an evolutionary system. Much depends on what the fact finder observes at the critical juncture in the evolution of the system, as illustrated by the circle in Figure 10.2. The point of departure within the circle can be changed by the internal dynamics of each firm in the system. Path A is the exponential evolutionary path, which may not be sustainable — it represents intense aggressive competition, with entry and exit of firms. It may, for example, characterise atomistic perfect competition in classical economics. However, once the surviving firms have established a level of market share, path B can better describe the evolution of a system more closely aligned to the evolution of markets in classical economics, from a starting position of competitive market, through the emergence of monopoly firms and an oligopoly structure capped at \( \bar{\beta} \).

In concentrated markets, where five or fewer players share 100 per cent market share, the zero-sum constraint allows each player to infer what the other player is prepared to sacrifice, and thus what they stand to gain by an action. Without knowing how much market share a player has, for example, a rival cannot really know whether an action on
price is meaningful or not. If the action on price can be described as a price dwarf, and if a rival is to react at time period t, then knowledge of market share is invaluable, because in its absence a reaction to such a price action could trigger a price war.

Technology has a long history that dates back many centuries. From the ox cart to the car, from the mainframe to the laptop, technology gaps are diminishing and consumer expectations are increasing. Many players are moving under the influence of each other’s technology pull. Some are moving unexpectedly fast as though being pulled by an invisible time-dependent set of preferences. The $\beta$ is defining the process of competition. The competitors are spherical competitors, since the technology allows competition from every angle. Observations of the signals, together with managerial theories about how management behave and have evolved, all point to Framework $Tn=3$. But the theory of firm behaviour combined with observations would suggest that $\beta$ is too lightweight to account for all competition that $\beta = n$. $[dT.dt]$ in that one can observe $\beta$ but cannot define it.

Figure 10.3 captures a stylised game pay-off between player A and player B on the level of price commitment. The pay-offs are computed by McNutt and Yang-Chan Hsu as illustrative of existing Nash equilibria in terms of the best one can do given the reaction of a competitor and the elusive $(4,3)$ that could be secured with price leadership – provided both players trust each other not to deviate from the agreed price leadership. The game dimension in terms of near rival may differ according to

**Figure 10.3**

Hsu-McNutt Signalling

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<tr>
<th>A signals high price</th>
<th>B reduces price</th>
<th>B reduces price</th>
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<th>B Signals High Price</th>
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<td>A Signals High Price</td>
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<td>(4,3)</td>
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<td>A Reduces Price</td>
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geography. For example, Nissan and Toyota are near rivals in Japan, but it is possible that in Europe the players could be Nissan and Volkswagen. Notwithstanding geography and type, an equilibrium requires a degree of trust and commitment.

Critical Reflective Thinking

In this book, we have argued that type is a function of signals; in other words, a signal is the first derivative of type with respect to time. At a point in time $t = T$, individual players learn from each other through signalling. No action is observed from a player until the player filters the information from the signal. In the case of a moon-shot (see Chapter 1), a player believes that the signal will translate into an action, thus he proceeds to act and is observed by other players. With signalling, a player stops and thinks; no action is observed at $t = T$, referred to as a do-nothing strategy. This differs from the mechanism of observational learning, where the action of one player is influenced by their observation of other players’ actions. McNutt (2005) commented that “Bayesian equilibrium does not take into account the fact that players may learn their opponents’ types by observing their play, since each move by a player may reveal information on his or her type (p. 92)”. In a Bayesian game, players have incomplete information about the characteristics of the other players. Management as players in the market-as-a-game can be described as participating in a Bayesian game and the signals on type are critical to playing the game. It is generally agreed amongst game theorists that players in a dynamic Bayesian game of incomplete information, players do learn their opponents’ types by observing their play, because each move by a player reveals new information on a player’s type. In the absence of new information, there is a challenge for a Bayesian market-as-a-game if others do not know, for example, that Player A in Figure 10.3 has betrayed his type — deceiving other players by signalling a high price but reducing price during the game. The Bayesian updating by other players relies on the observed action of Player A without reference to the formation or origin of the beliefs about Player A. Competitors should ask: why would a rival player be observed as doing nothing in a game? In a normal form Bayesian
game of incomplete information, the players are unable to update their prior beliefs on opponents’ types. One way to model incomplete information is to reduce the uncertainty in the game and convert it into a game of imperfect information. In this case, all Player A knows is his own type, and the fact that the other players who do not know his type observe his signals.

**Neo-Rational Action**

Learning from others in the market-as-a-game requires the players to understand what we refer to as the Humean neo-rational action of a player, McNutt (2010) — that is, a player betrays his type during a game by signalling X but doing Y. In other words, if management want to do Y (reduce price) and believe that doing X (signal a high price) is causally necessary to do Y then reason directs that they signal X in the game. If Player A does play Y and lowers the price, and this action is consistent to include observations by others of the action of playing Y, then playing Y is a neo-rational action by Player A: he may obtain a payoff of 5. In other words, Player A has fooled others into thinking that he is thinking about playing X prior to his decision to do so. He has created noise in the game by signalling X but doing Y — possibly to obtain a payoff of 5 or he may do nothing. The market-as-a-game may be a Bayesian game in the sense that information about the economic characteristics and type of other individuals in the game is incomplete but beliefs only role with respect to the observed action and conduct of Player A is to achieve a coherence with Player B’s intentions and the beliefs and preferences of other competitors about Player A. The neo-rational equilibrium could occur at payoff (4,3) for both players in Figure 10.3 if Player A is secure and Player B is thinking that Player A is thinking about playing X, and both players signal a high price, and a high price is obtained in the game.

**Blind Squirrels Find Nuts**

Bayesian equilibrium only takes into account the fact that players may learn their opponents’ **actions** by observing their play. The Bayesian
approach ignores a do-nothing strategy and it does not suggest a model on the origin of prior beliefs. In Framework Tn=3, beliefs are updated in the absence of new information. This arises because management have to focus on the origin of their respective belief about a competitor’s likely action. Management as players in the market-as-a-game observe their opponents’ types by observing their play but if no play or action is observed then management will learn-by-signalling. Hence, individual players are acting rationally at t = T when they do not ignore their own information about signals but do ignore the actions of others.

In Framework Tn=3, a signal can influence the action of a player in a game. For example, in Figure 10.3, if at t = K < T, player B signals a Baumol type by signalling sales revenue targets, this very strongly implies that player B’s prices will fall, but observing lower prices in a game at t = T does not necessarily mean that player B is a Baumol type. For example, if a new technology or functionality was added to player B’s product at t = K, creating an inelastic demand for the product, player B will increase price to maximise total revenues. Therefore, if the probability of lower prices in the absence of a Baumol type is greater than zero, the probability of lower prices with a Baumol type is less than one since the probabilities sum to one.

On the other hand, if lower prices did not happen when Player B was not a Baumol type, then observing that prices are lower at t = T in a game with Player B would always confirm that Player B is a Baumol type. Strategic reasoning is not the product of a very high probability that Y leads to X; that lower prices signal a Baumol type, but the product of a very low probability that not-Y (~Y) could have led to X. An error in the game occurs when too much attention is paid to p(X|Y) and not enough to p(X|~Y) when determining how much evidence X is for Y.

The degree to which a result X is evidence for Y depends, not only on the strength of the statement ‘we would expect to observe signal X if Y were true’, but also on the strength of the statement ‘we would not expect to observe signal X if Y were not true’. If Player B in Figure 10.3 is a Baumol type and always reduces price then it is rational for Player B to signal type and to obtain the payoff 4 in (3,4) provided Player A does not lower price. With that knowledge, Player A as a secure player
(Chapter 7) signals a high price trusting B to do likewise. For Player B, a 3 in payoff (4,3) is preferred to a payoff of 1 in (1,1).

Strategy is about player action and reaction; in an interdependent non cooperative game it is a rule telling management which action to choose at any given time. Game embedded strategy is about knowing when and knowing how to act. Interdependence is recognised, beliefs are formed and management rethink the strategy of ‘going it alone’ in order to obtain a competitive advantage in the market. Heraclitus, a Greek philosopher of the sixth century BC, wrote, “Character is destiny”; Framework Tn=3 recognises that management type is destiny, and that understanding type, coupled with understanding technology and time, is intricately linked to sustaining a competitive advantage in the market-as-a-game.