Practical III

War Games: Constructing Nash Reply Functions & Computing a Nash Equilibrium

Patrick McNutt

www.patrickmcnutt.com

In this *Aide Memoire*, we focus on the end point of a price game. By computing a Nash equilibrium as the end point in a Bertrand price game we can offer strategic advice to a client preparing a playbook. A set of price data is provided, a selected set of price points used in a 'what-if' war game scenario simulation on behalf of Client B, who is contemplating entry into a Bertrand price game. We develop a war-game scenario based on price data.

War games are part of management decision making and check out this **McKinsey Quarterly**:

https://www.mckinsey.com/business-functions/strategy-and-corporate-finance/our-insights/playing-war-games-to-win

Player A is a dominant incumbent aced with the threat of entry at the EPOS[1] price point 699. A strategy playbook was opened in order to add value to management's decision making – enter with risk-off symmetric move. We advise on a simulation of a Bertrand price sequence in the likely event of a mismatch in the signaling.

Client consultation:

You are retained by player B, the entrant player, to advise on how to play the game.

Should player B enter at a price just below Pm to initiate a game reaction from player A? Think like player A: if you were advising player A would you

[1] Electronic point of sale = it is the price recorded on the consumer's receipt of purchase. Also recorded by the store's information system.

recommend to player A to trust player B and end the game early in the sequence?

If not, then begin to simulate the price sequences of both players in order to compute the NE.

Playbook

Identify the near-rival as that competitor who is more likely, with a degree of probability, to react first on entry with price movements. Assess the near-rival's type as defined by the number of moves, the magnitude of price moves and the frequency of price moves. Historic price moves are assigned to a Fibonacci sequence. This is a two-player game, a game between A v B.

In any 'what-if' simulation an opening move is required and an end point: A's price point 699 is defined as the opening move and we deployed the NE concept to define an end point. In the Masterclass the executives will be presented with the geometry of the analysis. In the interim, readers should look at Figure 9.4 pp143 in McNutt's *Decoding Strategy*.

Corollary: Note the intersection of Nash reaction functions; the point of intersection provides a NE point, an end point in the game. The end point is analogous to the blowing of a referee's whistle that end a sports game after a discrete finite time period.

Player Type

Player type as defined by the move patterns and we ascribed a Turing pattern to the game by focusing on the early sequence of moves. We assign moves to the Client B and the near-rival A. The manual construction of the NE as an end point in the game is the class exercise and it simply provides an illustration of a 'what-if' T/3 simulation game.

https://www.amazon.com/Decoding-Strategy-Predictions-Patrick-McNutt/dp/1259071065

Scenario Planning

Different information on type and new corporate intelligence furnishes a second and a third simulation round. This provides confirmation that should B enter the game there is a non-zero probability of a Bertrand price war of at least 5 moves duration converging towards Nash equilibrium end point.

Different Scenarios are simulated in order to provide Client B with a playbook and an optimal strategy set with a payoff-dominant outcome. More corporate intelligence is gathered to support a data-driven strategy coupled with C-suite cycle of non-lateral strategic thinking guided by NORA and OODA loop[2] in order to decode strategy in the game.

Note: Computing and Finding NE

1. Learning curve: play the game and collect information during the game. The risk is the credibility of the signals, noise and the Bayesian persuasion problem of sender's signal influencing the action of the recipient.
2. Backward induction: Construct an extensive decision tree allowing the branches of the tree to reflect corporate intelligence on players' type and rival behavior. At each node of the tree we assign a payoff number, normalized to 0 if no entry to the game and the pair (-1,1) to reflect relatively higher payoffs.
3. Prisoners' dilemma: Transfer the payoff numbers into a normal payoff matrix and find the NE payoff in the matrix (*vide* Lecture Masterclass Notes)
4. Turing geometry: Convert type into a Turing pattern of moves as in class handout and construct the Nash reaction functions and find the point of intersection.
5. Link to Diagram in textbook.

Game Dimension

There are two players in a game G, player A and player B. Data analysis has already established that B is the near rival to A by (i) interrogation of the data patterns; (ii) computing inter-brand cross-price elasticity, (iii) filtering belief systems into the CV matrix.

Opening price signal: 699

Player A's type & playbook

From A's historic pattern sequences the **first move of A** is likely to be a price reduction of magnitude **ΔP = 29%** of Pm followed in sequence by a **2nd move of ΔP = 29%** reduction on rival price and a **3rd move of a ΔP = 14%** price

[2] OODA refers to the cycle of observe, orient, decide and act in strategic decision making. NORA refers to the non-obvious trends in the data patterns.

reaction leading to a cumulative price reduction from the opening move at price Pm. In a dynamic market-as-a-game, player B could interpret A's 1st move as an entry deterrent move. If player A has a reputation for ΔP from an opening 1st move of 29% and if *pre-entry* player B believes that this will be the likely reaction of A, post-entry, player B may be persuaded not to enter. An example of limit pricing and Bayesian persuasion: should B believe the signals from A?

Player B's type & playbook

B is not interested in an expensive price war of infinite duration. In the *I-think-you think-I think* strategy player A believes that B's price reactions in a Bertrand game will follow a pattern beginning with an opening symmetric move of match-match with **1st move ΔP = 2%** differential on Pm as the opening move by B to be interpreted as no interest in a price war. However, A also believes that if B keeps to type that **B's 2nd move ΔP = 36%** and **3rd move with magnitude ΔP = 27%**. Playing this sequence of moves demonstrates B's *commitment* to playing the Bertrand game to win.

Playbooks: B to move first as the entrant

	A	B	
		2%	1st move
1st move	29%	36%	2nd move
2nd move	29%	27%	3rd move
3rd move	14%		4th move
4th move	....	....	
	....	....	
nth move			nth move

However, player B is known to *camouflage* its move sequence in order to create *noise* in the game. With noise in a game player A will think that the opponent B by reacting with a price signal ΔP = 36% in a 2nd move is signaling a strategy of hit-and-run end to the game with a punishment move since B's 36% > A's 29%. In any war game simulation, any player can signal an end to the game before the nth move. If the opponent misreads the signal and there is a mismatch the game will continue until a Nash equilibrium is reached assuming that NE is reached at the nth move but rational players will exit or end the game at (n-1)th move.

Corporate Intelligence

The following exercise allows you to compute the NE price and to advise the client on entry by price signaling in the neighborhood of Pm and to signal an end to the game by ΔP before NE is reached. Your strategic advice may be that both players would be better off if they opted to play a Cournot game and compete for the market.

Further 'war-game' scenario planning will require additional information on player type. Could Player A be a Stackelberg-leader in this game? Does A have a *reputation* for using a *fighting ship* if threatened by a price competitor. Player B is a *de novo entrant* but continues to *camouflage* its reserve capacity. What if Player B is the *fighting ship* of a spherical competitor C not yet in the game at time period t but every intention to enter at t+1?

Class Exercise: Scenario Planning and Simulation

With information on player type and with the information on likely price sequences fill in the simulation box below and proceed to chart the likely Nash reaction functions. Player A is an incumbent type and B is a de novo entrant type.

Draw a diagram with Player A's price moves on the vertical axis and Player B on the horizontal axis. Given that NE is where the best reply reaction functions intersect:

1. Allow for a NE to be reached by magnitude ΔP at the 8^{th} move in the sequence.
2. Player A's opening price is Pm = 699, player A expects B to enter with (i) symmetric move and/or (ii) player B will follow in the price sequence if both players have a CV ≠ 0.
3. B's 4^{th} move = B's 1^{st} move ΔP = 2% signaling the game end point at a best reply NE equilibrium price point = 139.
4. Apply Turing sequence – focus on the early moves, two moves per player in order to draw a diagram.

Nash equilibrium (Convergent) End move = 139

The sequence of player moves in this *Bertrand* price game is as follows: player B enters at the initial move of *match-match* however the move triggers a price reaction from A:

Game opens at 699. If B plays match-match without provoking a price reaction from A then B would enter at 699 or 'shading' price by 1% differential at (699 – 6.99 = 692). However, if B keeps to type with 1st move ΔP = 2% then we are looking at the playbook with 699 and 686……..into the simulation box below.

B's 699 is match-match move

B's 692 is a shading price move

B's 686 triggers a price reaction from A.

T/3 Simulation Box

A	B	A	B	A	B	A	B	NE
Pm = ***699*** $CV^A \neq 0$ **START**	B 1st move	A 1st move	B 2nd move	A 2nd move	B 3rd move	A 3rd move	B 4th move	A 4th Move = B 4th move **END**

Ends//